Problem-Solving & Reasoning

Cognitive Development
"Kids are weird"

<table>
<thead>
<tr>
<th></th>
<th>Smart</th>
<th>Not so smart</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Perceptual Development</strong></td>
<td>don’t fall for perceptual illusions predicted by philosophers</td>
<td>do fall for illusions that would embarrass an adult</td>
</tr>
<tr>
<td><strong>Language Development</strong></td>
<td>learn phonology and syntax better than adults</td>
<td>babble non-native phonemes; too rule-governed: “breaked”, “goed”</td>
</tr>
<tr>
<td><strong>Concept Development</strong></td>
<td>organize experiences into concepts encoding object categories, number of a set, locations of objects, &amp;c</td>
<td>too statistics-governed; logarithmic; ignore landmarks, &amp;c.</td>
</tr>
</tbody>
</table>
Children’s Problem-Solving
Seven Sins of Children’s Problem-Solving

1. Inhibiting action is difficult
2. Inhibiting thought is difficult
3. Over-optimism
4. Foiled planning
5. Limited encoding of problems
6. Limited encoding of solutions
7. Failure to integrate
1) Inhibiting Action
1) Failure to Inhibit Action

- **Visual Search**: Three-year-olds also search towards the location where they last found the object (Schutte & Spencer, 2002)
- **Choice Reaction Tasks**: Even adolescents have difficulty inhibiting previously successful actions (stop signal task: Logan, 1994; Band et al., 2000)
1) Inhibiting Action

According to the hypothesis that a global mechanism underlies changes in RT, the ratio between task RTs should be almost equivalent for children and adult age groups. However, the difference between activation and inhibition latencies was much larger for child groups than for the adult group. Analysis of variance performed on the individual ratios between the estimated inhibition times and simple RTs (e.g., \(\frac{\text{stop-all latency}}{\text{simple RT}} / \text{simple RT}\)) yielded significant main effects of age group, \(F(3, 54) = 11.54, p < .0001\), for the stop-all versus simple RT comparison and, \(F(3, 54) = 8.15, p < .0001\), for the stop-change versus simple RT comparison. The ratios increased with age for both comparisons, indicating that developmental change in response activation, indexed by simple RT, is more pronounced than age differences in the speed of inhibition, indexed by stop-all and stop-change latencies. Finally, a similar ratio analysis was performed including participants' mean simple and choice RTs. The analysis of variance performed on these ratios failed to reveal a significant main effect of age group, \(F(3, 54) = 1\). This finding provides additional support for the hypothesis that response activation and response inhibition follow different developmental trajectories.

FIG. 6. Brinley scatter plot of the data presented in Figs. 1 and 5. Data points combine the mean reaction times (in milliseconds) for young adults on a given task condition with the corresponding reaction times for a child group. Activation latencies are plotted in white, and inhibition latencies are plotted in black. Lines represent the best fitting linear regression functions through the eight data points of each child group, with \(r^2\)'s of .341, .800, and .912 for the 5-, 8-, and 11-year-olds, respectively. Note the poor fit of the linear equation for young children.
1) Inhibiting Action

Delay of Gratification
2) Inhibiting Thought

Figure 2 | Sample target cards and test cards used during the pre-switch phase of the DCCS (top panel). The bottom panel shows the target cards and test cards used during the post-switch phase for several versions of the DCCS (based on the assumption that color is the initial, pre-switch dimension).
2) Inhibiting Thought

- **Stroop Effect**: Even as adults, we have difficulty inhibiting previously relevant thoughts.

<table>
<thead>
<tr>
<th>Read word</th>
<th>Read color</th>
<th>Read color</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>XXX</td>
<td>green</td>
</tr>
<tr>
<td>green</td>
<td>XXX</td>
<td>blue</td>
</tr>
<tr>
<td>blue</td>
<td>XXX</td>
<td>black</td>
</tr>
<tr>
<td>black</td>
<td>XXX</td>
<td>red</td>
</tr>
</tbody>
</table>
2) Inhibiting Thought

- **Stroop Effect:** Beginning readers > Expert readers
- **Addition Equivalence:** U-Shaped pattern

Table 1

<table>
<thead>
<tr>
<th>Solution</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Correct</td>
</tr>
<tr>
<td>25</td>
<td>Add all</td>
</tr>
<tr>
<td>4</td>
<td>Carry</td>
</tr>
<tr>
<td>16</td>
<td>Add to equal sign</td>
</tr>
</tbody>
</table>

Figure 2. Proportion of children who solved at least one equivalence problem correctly as a function of age.  
$y =$ years; m = months.
Inhibiting Thought & Action
Implications for Perspective Taking

1. Inhibition proficiency is highly correlated with performance on “theory of mind” tasks that require taking others’ perspectives into account (Sabbagh et al., 2006), rely on overlapping cortical areas (Sabbagh, 2004), and these areas do not reach maturity until early adolescence (Goldman-Rakic, 1994)
Probably for this reason, 7-year-olds who do well at taking one other perspective into account do much worse at taking multiple perspectives into account (Knight et al., 1995, 1997)
Seven Sins of Children’s Problem-Solving

1. Inhibiting action is difficult
2. Inhibiting thought is difficult
3. **Over-optimism**
4. Foiled planning
5. Limited encoding of problems
6. Limited encoding of solutions
7. Failure to integrate
3) Over-optimism

Figure 1. Schematic representation of vertical reach, horizontal reach, stepping, and clearance tasks used in Experiments 1 and 2.

Figure 2. Mean percentage of correct judgments as a function of age and difficulty level in Experiment 1.

Plumert (1995)
4) Foiled planning

- Planning can be a waste of time and effort for children
  - Children often fail to execute good plans correctly (Berg, 1989; Schauble, 1996)
  - Children’s plans often depend on others’ cooperation, but children frequently bicker, lose track of the original task, and refuse to cooperate (Baker-Sennet et al., 1992)
  - Failed plans often bring no consequences to children (e.g., parents helping procrastinating children with their last minute projects; Ellis et al., 1992)
Seven Sins of Children’s Problem-Solving

1. Inhibiting action is difficult
2. Inhibiting thought is difficult
3. Over-optimism
4. Foiled planning
5. Limited encoding of problems
6. Limited encoding of solutions
7. Failure to integrate
Failures of encoding

- **Balance Scale Problem** (Siegler, 1976)
  - **Rule 1**: If the weight is same on both sides, predict balance; otherwise, side with more weight goes down.
  - **Rule 2**: If one side has more weight, predict it will go down. If weights on two sides are equal (Problem A), choose side with greater distance.
  - **Rule 3**: If both weight and distance are equal, predict balance. If one side has more weight or distance, and two side are equal on other dimension, predict that side with greater value on unequal dimension will go down. If one side has more weight and other more distance, guess (Problem B).
  - **Rule 4**: Multiply weight times distance (torque). Predict side with greater torque goes down.
# Failure to integrate dimensions

<table>
<thead>
<tr>
<th>Task</th>
<th>A. Dominant dimension</th>
<th>B. Subordinate dimension</th>
<th>C. Relation between A &amp; B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance scale</td>
<td>Weight</td>
<td>Distance from fulcrum</td>
<td>( C = A \times B )</td>
</tr>
<tr>
<td>Conservation of liquid</td>
<td>Height of liquid</td>
<td>Cross-sectional area of liquid</td>
<td>( C = A \times B )</td>
</tr>
<tr>
<td>Conservation of number</td>
<td>Length of row</td>
<td>Density of objects in row</td>
<td>( C = A \times B )</td>
</tr>
</tbody>
</table>
Lemonade Stand Task

- Predict what would happen to the other three variables if there were a change in  
  - demand for lemonade (e.g., if it was hotter than normal)  
  - supply for lemonade (e.g., if there were more lemonade stands)  
  - price of lemonade (e.g., if lemonade were offered for less)  
  - sales of lemonade (e.g., if more lemonade was sold)
Lemonade Stand Task

- To solve these problems, students must encode
  - lemonade makers’ motivation for profit
  - competition among lemonade makers
  - lemonade consumers’ motivation for lemonade
  - lemonade consumers’ motivation to economize
## Failures of encoding

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Grade</th>
<th>Profit Seeking</th>
<th>Economizing</th>
<th>Competition</th>
<th>Acquisition of Desired Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit seeking</td>
<td>K</td>
<td>13</td>
<td>74</td>
<td>74</td>
<td>10</td>
</tr>
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<td></td>
<td>2</td>
<td>51</td>
<td>77</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economizing</td>
<td>K</td>
<td>17</td>
<td>77</td>
<td>80</td>
<td>15</td>
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<td>2</td>
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<td>80</td>
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<td></td>
<td>4</td>
<td></td>
<td></td>
<td>80</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Competition among sellers</td>
<td>K</td>
<td></td>
<td></td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>55</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>Acquisition of desired goods</td>
<td>K</td>
<td></td>
<td></td>
<td></td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Implications for Connected Learning

1. **Children often have more difficulty understanding motivations of sellers vs buyers**
2. **Fail to encode relevant relations among stakeholders**
3. **When they do encode these relations, they often fail to integrate them appropriately**
Where do children go right?

Solutions are **creative**

Children see new problems as *analogous* to old problems for which they have solutions

Q: When do analogies occur to children?
Stimulus Sets

Cross-Mapping

Literal Similarity

FIG. 6.4. Sample stimuli displaying the monotonic change relation from the mapping studies (Gentner & Rattermann, 1991; Rattermann & Gentner, 1990; Rattermann et al., 1994). Two of the objects are cross-mapped for both the (a) sparse and (b) rich pairs.
FIG. 6.5. Proportion of relational responses for different age groups in each condition of the mapping study (Gentner & Rattermann, 1991; Rattermann & Gentner, 1990; Rattermann et al., 1990).
Effect of Labels

Cross-Mapping

Sparse

Daddy  Mommy  Baby

Effect of Labels

Two of the objects are cross-mapped for both (a) sparse and (b) rich pairs. The easy pairs were literal similarity pairs, in which the object similarities supported the required relational alignment; the difficult pairs were cross-mapped pairs, in which the object similarities opposed the relational alignment (Genner & Rattermann, 1991; Rattermann & Genner, 1990; Rattermann et al., 1994).

As in the study by Markman and Genner described earlier, we varied the richness of the objects so that the interplay between object similarities and relational similarities could be examined.

Children were presented with two configurations of objects, each ranged according to the monotonic increase (or decrease) in size relation, operationalized as three objects in a row, increasing in size from left to right or right to left (see Fig. 6.4). One set of objects was designated as the child’s (C) set, the other as the experimenter’s (E) set. The child was asked to close his eyes while the experimenter hid stickers under one object in each set.

"For clarity of presentation, we assume a male subject and a female experimenter, although in fact, roughly equal numbers of boys and girls participated.

Daddy  Mommy  Baby

Daddy  Mommy  Baby

Daddy  Mommy  Baby
We showed 4-, 6-, and 8-year-old children (12 children at each age) trials of figures and asked them to say which of two alternatives was most similar to the standard. The figures were sets of squares or circles differing in size and darkness. The standard was always constructed to fit one of two higher order perceptual relations: either monotonic change or symmetry. Monotonic change was operationalized as three objects in a line, identical except for the dimension of interest—either size or darkness (see Fig. 6.6).

Although the child could select either response—there was no feedback—one of the two choices was always clearly more similar to the standard from the adult point of view. This alternative, the relational choice, depicted the same relational structure as the standard, but contained different objects.

The other comparison figure (the foil or non-relational choice) used the same objects as the relational choice, but these objects were haphazardly arranged so that there was no good higher-order relational structure. Thus, both alternatives matched the standard equally well at the object level, and the relational alternative matched better at the relational level, making the relational match preferable to anyone who recognized the matching relational.
TABLE 6.1
Mean Proportion of Relational Responses by Age and Condition

<table>
<thead>
<tr>
<th>Age</th>
<th>Same-Polarity</th>
<th>Opposite-Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Same-Dimension</td>
<td>Cross-Dimension</td>
</tr>
<tr>
<td>4</td>
<td>0.68</td>
<td>0.49</td>
</tr>
<tr>
<td>6</td>
<td>0.90</td>
<td>0.75</td>
</tr>
<tr>
<td>8</td>
<td>0.96</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Progressive Alignment

Easy Align to Hard Align
Thompson & Opfer (2009)

- Test: Gave 64 7-year-olds training on 0-100 number line problems; given 0-1,000, 0-10,000, 0-100,000 number lines
- To elicit analogies in 1/2 of the children:
  - Alignment group: Aligned the 0-100 number lines with the 0-1000, 0-10000, 0-100000 number lines, and highlighted perceptual similarity of the types of problems
  - No Alignment group: the number lines were presented with the same perceptual similarity but with no alignment
Sample Training Problems
Sample Training Problems

0  -  15  -  100
Sample Training Problems

[Diagram showing a number line from 0 to 100 with a mark at 15]
Sample Training Problems
Alignment Group

Training Set

Generalization Set

15

0 100

15

0 100

15 150

0 100 1000

15 1500

0 100 10000

15 15000

0 100 100000
No Alignment Condition

Training Set

15

0 100

Generalization Set

15

0 100

150

0 1000

1500

0 10000

15000

0 100000
Predictions

No Alignment

Much greater errors;  
Much more logarithmic

Alignment

Not much greater errors;  
not much more logarithmic
Accuracy

Training Set

Generalization Set

- No Alignment
- Alignment
Generalization Problems

No Alignment

<table>
<thead>
<tr>
<th>Number</th>
<th>Median Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>0</td>
</tr>
<tr>
<td>0-1000</td>
<td>25</td>
</tr>
<tr>
<td>0-10000</td>
<td>50</td>
</tr>
<tr>
<td>0-100000</td>
<td>75</td>
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\[ R^2_{\text{lin}} = 1.0 \]

<table>
<thead>
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<tbody>
<tr>
<td>0-100</td>
<td>0</td>
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<tr>
<td>0-1000</td>
<td>250</td>
</tr>
<tr>
<td>0-10000</td>
<td>500</td>
</tr>
<tr>
<td>0-100000</td>
<td>750</td>
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\[ R^2_{\text{log}} = .90 \]

High Alignment

<table>
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<tbody>
<tr>
<td>0-100</td>
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<td>0-1000</td>
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<td>50</td>
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<td>0-100000</td>
<td>75</td>
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\[ R^2_{\text{lin}} = .99 \]

<table>
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<td>0-1000</td>
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<tr>
<td>0-10000</td>
<td>500</td>
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<td>0-100000</td>
<td>750</td>
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</table>

\[ R^2_{\text{lin}} = .95 \]

<table>
<thead>
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<td>0-10000</td>
<td>5000</td>
</tr>
<tr>
<td>0-100000</td>
<td>7500</td>
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\[ R^2_{\text{log}} = .85 \]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>0-100</td>
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<td>0-10000</td>
<td>50000</td>
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<tr>
<td>0-100000</td>
<td>75000</td>
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\[ R^2_{\text{log}} = .86 \]

<table>
<thead>
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<tr>
<td>0-10000</td>
<td>500000</td>
</tr>
<tr>
<td>0-100000</td>
<td>750000</td>
</tr>
</tbody>
</table>

\[ R^2_{\text{log}} = .85 \]
Representational Change

- **Basic Idea:** Analogy provides a powerful mechanism of representational change
- Development of the mental number line
  - Alignment led to higher accuracy and more adult-like representations outside *(far outside!)* the training set
  - This broad generalization didn’t happen because the *kids* in the alignment group were especially smart—it happened because a *good analogy* *(2.apple:20::6.apple:60)* made them smart
Study 2

- If smart analogies make us smart, maybe dumb analogies make us dumb...
- Maybe progressively aligning problems with mismatching structures will lead us to generalize the *wrong* structure...
- **Test:** Effect of notation on estimates of real numbers
  - College students and second graders
    - Some saw real #s expressed in decimal notation (good analogy to whole numbers)
    - Some saw same real #s expressed in fractional notation (bad analogy to whole numbers)
“If this is where $1 every minute goes, and this is where $1 every 1,440 minutes goes, if a person makes $1 every 60 minutes, where does that go on this money line?”

(Opfer & DeVries, 2008)
60 minutes

1 minute 1440 minutes

$1 60 minutes

$1 1 minute 1440 minutes
BAD ANALOGY

1 minute

Adults

60 minutes

Kids

1440 minutes

$1

1 minute

Adults

$1

60 minutes

Kids

$1

1440 minutes
If kids and adults draw analogy b/w natural and real numbers, children’s estimates would increase logarithmically w/ denominator, whereas adults’ would increase linearly.

(Opfer & DeVries, 2008)
Fraction Notation

2nd Graders

Adults

Log $R^2 = 0.99$

Close approximation to correct performance (power function)

Lin $R^2 = 0.94$

Not as close approximation to correct performance (power function)

( Opfer & DeVries, 2008)
Fractional Notation

Accuracy

![Graph showing the accuracy of Fractional Notation for 2nd Graders and Adults.](Opfer & DeVries, 2008)
• Implication is NOT that kids are better at fractions than adults
  • Numerals (1, 2, 3, ...) automatically activate an analog magnitude representation that stores numerosity information (.apple, apple, apples) in memory
  • Numerals in fractions, therefore, are misleading to adults and children—but not equally misleading

• If real numbers could be expressed using the same notation used for natural numbers, adults should do better than kids
Decimal Notation

$0.0167$ per minute

$\frac{1}{50}$ per minute

$0.0007$ per minute
Decimal Notation

- **Log R^2**:
  - 0%
  - 25%
  - 50%
  - 75%
  - 100%

- **Decimal Value Given**
  - 0% to 1

- **Decimal Notation**
  - **Log R^2** = .99
  - **Lin R^2** = .82
  - **Lin R^2** = .94

- **Median Distance of Estimate from Origin**
  - 0 - 1000 Number line (Correct Performance)
  - 0 - 1000 Number line (Child Performance)
  - 1/1 - 1/1440 Fraction Line (Correct Performance)
  - 1/1 - 1/1440 Fraction Line (Child Performance)
  - 1/1 - 1/1440 Fraction Line (Adult Performance)
  - $1 per minute - $1 per day Fraction Line (Correct Performance)
  - $1 per minute - $1 per day Fraction Line (Child Performance)
  - $1 per minute - $1 per day Fraction Line (Adult Performance)
Decimal Notation

To examine this relation, we regressed magnitude estimates. To examine this relation, we regressed magnitude estimates against the value of the denominator using a linear representation and a logarithmic representation. As predicted, the fit of the linear function to participants’ estimates against the value of the denominator was better than the fit of the logarithmic function. Due to the difficulty of the computational estimation involved, children and adults did not capitalize on the semantic knowledge the task offered. As in the common numerators task, adults and children not only provided a better fit than the logarithmic function, but the linear function also fit children’s estimates as in the common numerators task (cf. Figs. 3 and 4). Last, whether adults’ familiarity with units such as hour, day, and minute might improve their performance. Rather, adults’ performance was similar to performance on the logarithmic function in minutes. As predicted, the fit of the linear function to their median estimates against the value of the denominator was better than the fit of the logarithmic function.

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Summary

1. **Tendency for a relational shift**: Older children choose relational matches more often than younger children.

2. **Shift seems to arise from knowledge**. When relations are well-known to younger children, they also make relational matches.

3. **Use of relational language promotes relational mapping**.

4. **Because structural alignment induces attention to relational structure**, **comparison process promotes relational mapping**.

5. **Because comparison process is multiply constrained**, both **object similarity and relational similarity affects performance**.