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Brief article

Representational change and magnitude estimation: Why young children can make more accurate salary comparisons than adults ☆

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ABSTRACT

Development of estimation has been ascribed to two sources: (1) a change from logarithmic to linear representations of number and (2) development of general mathematical skills. To test the representational change hypothesis, we gave children and adults a task in which an automatic, linear representation is less adaptive than the logarithmic representation: estimating the value of salaries given in fractional notation. The representational change hypothesis generated the surprising (and accurate) prediction that when estimating the magnitude of salaries given in fractional notation, young children would outperform adults, whereas when estimating the magnitude of the same salaries given in decimal notation, adults would outperform children.

1. Introduction

Although estimation plays a central role in many activities, children's estimates are highly inaccurate (see Siegler & Booth, 2005, for review). This difficulty has been ascribed to a number of sources, including development of numerical representations (Joram, Subrahmanyam, & Gelman, 1998; Siegler & Opfer, 2003) and general improvements in mathematical skills (Dowker, Flood, Griffiths, Harriss, & Hook, 1996; Noël, Rouselle, & Mussolin, 2005). To test these explanations, we examined development of fractional magnitude estimation, where the representational change hypothesis generated the surprising prediction that young children would make more accurate estimates of fractional magnitudes in some situations than would adults.

According to the representational change hypothesis, children's inaccurate estimation stems from reliance on logarithmic representations of numerical value (Siegler & Opfer, 2003). For example, asked to estimate locations of numbers on number lines with 0 at one end and 1000 at the other, almost all second graders tested by Siegler and Opfer (2003) produced logarithmic distributions of estimates (e.g., estimating 150 to be closer to 1000 than to 1), suggesting that children initially use a representation of numerical value that is consistent with Fechner's Law and that is widespread among species, human infants, and time-pressured adults (see Dehaene, Dehaene-Lambertz, & Cohen, 1998, for review). In contrast, all adults in the same study produced linear distributions of estimates (e.g., estimating 150 to be closer to 1 than to 1000), and about half of fourth graders produced estimates that were best fit by each function (Siegler & Opfer, 2003), suggesting that the use of a linear representation of number becomes more widespread with age and experience. With younger children and lower numeric ranges, this developmental change from logarithmic to linear numeric estimation has been replicated several times (e.g., Opfer & Siegler, 2007; Siegler & Booth, 2004) and shown to correlate with a wide range of other estimation tasks (Booth & Siegler, 2006; Laski & Siegler, 2007).

Although evidence of children's poor estimation skills and logarithmic representation of numeric value has been

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drawn from a wide range of tasks, later accuracy in numerical estimation could be attained either from representational changes (e.g., switching from a logarithmic to linear representation of numerical value) or from improving mathematical skills (e.g., learning division would enable students to estimate the number of partitions to make on a number line task). To address this issue, we sought to provide a particularly strong test for the representational change hypothesis by examining development on estimation tasks that favor the logarithmic representation over the linear one. We reasoned that if people learn to make better estimates simply by improving general mathematical skills, estimation on such tasks should also improve. However, if people learn to make better estimates by automatizing use of a linear representation, estimation on these tasks should suffer with age and experience.

One case in which an automatic linear representation of numerals would be misleading is in comparing fractional magnitudes that share a numerator. Specifically, we propose that adults' success in using a linear representation of number has an unintended negative consequence: by automatizing that 150 is closer to 1 than to 1000, adults are subject to a powerful cognitive illusion in which 1/150 seems closer to 1/1 than to 1/1000; in contrast, children's belief that 150 is closer to 1000 than to 1 protects them from this illusion. Thus, if the representational change hypothesis is correct, young children – despite their poor understanding of fractions and poor estimation skills generally – would make more accurate estimates than adults when asked to estimate the magnitude of fractions.

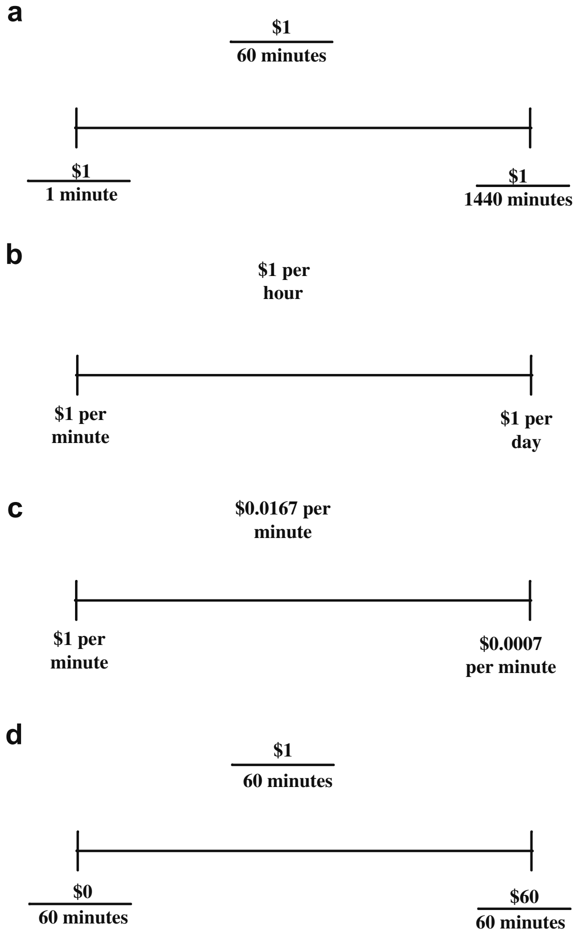

Fig. 1. The tasks: (a) common numerators, (b) familiar units, (c) decimal units, and (d) common denominators.

1.1. Estimation of fractional magnitude

Far from suggesting that children would have an advantage in estimating fractional magnitude, fractions typically pose enormous difficulties for children and adults (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981). This difficulty arises largely because the magnitudes of the denominator and numerator – but not the magnitude of the fraction itself – are represented automatically. Thus, when estimating the answer to 12/13 + 7/8, fewer than a third of 13- and 17-year-olds correctly chose "2" from the options "1", "2", "19", and "21" (Carpenter et al., 1981). Had students represented the magnitude of 12/13 and 7/8 as each being about equal to 1, the answer would have been easy to solve (1 + 1 = 2). Instead, about half of students answered 19 or 21, indicating that they focused exclusively on numerators (12 + 7 = 19) or denominators (13 + 8 = 21).

This interpretation of young adults' approach to fractions led us to make two predictions about how they would estimate the value of quantities expressed in fractional notation, such as when estimating the placement of a salary (e.g., $1/60 min) on a line that begins with one salary (e.g., $1/min) and ends with another (e.g., $1/1440 min) (see Fig. 1). The first prediction was that if adults compare only the value of the denominators in the salary, their linear representation of number would lead to radically inaccurate estimates. This inaccuracy is predicted by the fact that the relation between the numeral expressed in the denominator and the magnitude denoted by the whole fraction is provided by a power function rather than by a linear function. For example, although 60 is closer to 1 than to 1440, $k/60$ is closer to $k/1440$ than it is to $k/1$.

The second prediction was that if children also compare denominators, their logarithmic representation of numbers would have a correcting effect and thereby lead to more accurate estimates than those of adults. This prediction stems from the fact that the power relation between the value of the denominator and the magnitude of the fraction is somewhat similar to that of a logarithmic function. Thus, for example, the natural logarithm of 60 is closer to the natural logarithm of 1440 than to the natural logarithm of 1, much as $k/60$ is closer to $k/1440$ than it is $k/1$.

Although the combination of comparing denominators and having a logarithmic representation of number could lead children to outperform adults, this is not a trivial prediction: "'Performance improves with age' is as close to a law as any generalization that has emerged from the study of cognitive development" (Siegler, 2004, p. 2). In the present case, several cognitive processes could lead adults to make more accurate estimates, including thinking about stimuli in terms of money divided by time (e.g., converting salaries such as \$1/2 min into decimal equivalents such as \$0.50/min), thinking about stimuli in terms of real-world equivalents (anchors) and adjustments from those anchors (e.g., calling on real-world knowledge that people making only \$1/day and \$1/h are both much poorer than people making \$60/h), or reframing stimuli so they possess common denominators and then simply comparing the numerators (e.g., by converting the problem of placing 1/60 on a 1/1–1/1440 scale to the problem of placing 24/1440 on a 1440/1440–1/1440 scale).

In the present study, we addressed children's and adults' ability to apply these four different ways of thinking about salary magnitudes by comparing the accuracy of salary judgments when the salaries were expressed in terms of decimals, familiar units, common denominators, and common numerators. Hypothesizing that the mental representation of numerals themselves was the most important variable in how people estimate the magnitude of salaries, we predicted that children would outperform adults when given information in terms of common numerators and familiar units, whereas adults would outperform children when given information in terms of decimals and common denominators.

2. Method

2.1. Participants

Participants were 24 second graders (mean age = 8.4 -years, *SD* = 0.40) and 24 undergraduates (mean age = 19.4 -years, *SD* = .77). An additional 11 undergraduates completed the common denominators control task.

2.2. Tasks

Across all tasks, participants were asked to estimate the total money a person would make at a given salary (e.g., \$1/h) by placing a mark on a "money line" (see Fig. 1). The value of salaries to be estimated was held constant across the tasks, but we varied notations for expressing salaries (see Table 1). On the common numerators and common denominators tasks, the value of the salaries to be estimated (e.g., \$1/60 min) were expressed in fractional units, as were the endpoints of the money line (common numerator: \$1/1 min, \$1/1440 min; common denominator: \$0/*n* min \$*n*/*n* min, where *n* corresponded to the denominator to be estimated). On the familiar units task, salaries and endpoints were expressed in familiar units (e.g., \$1/h, \$1/min, \$1/day). On the decimal units task, salaries and endpoints were expressed in decimal units (e.g., \$0.0167/min, \$1/min, \$0.0007/min).

2.3. Design and procedure

Participants were told they would be given "money lines" representing the value of different salaries. For each problem, the experimenter indicated the two endpoints of the money line, indicated the target salary above the line, and asked the participant to make a hatch mark on the line to indicate its value. For example, in the familiar units task, the participant was asked, "If this is \$1/min and this is \$1/day, where would \$1/h go?" Order of tasks was counterbalanced over subjects, and order of problems within each task was randomized. Half of participants solved problems with the smaller salary on the left and vice-versa.

3. Results and discussion

We first converted all estimates (i.e., hatch marks) to a numeric code by measuring the distance between the scalar origin and estimate (0–25 cm) and dividing by total scalar length (25 cm). To assess accuracy, we examined the mean absolute error for each subject, which was calculated using the formula, where *n* equals the number of trials in a block,

$$\text{MAbsErr} = \sum_{i=1}^{n} \frac{|\text{actual}_i - \text{estimate}_i|}{n}$$

We then conducted a 2 (age group: 8-year-olds, adults) × 3 (task: common numerators, familiar units, decimal units) repeated-measures ANOVA on mean absolute error (MAbsErr) scores for each subject (see Fig. 2, where accuracy reflects the inverse of MAbsErr). As predicted, 8-year-olds (MAbsErr, *M* = .44, *SD* = .15) provided better estimates than adults (MAbsErr, *M* = .50, *SD* = .27), $F(1, 138) = 6.86$, $p < .01$, and the decimal units task elicited greater accuracy (MAbsErr, *M* = .27, *SD* = .16) than the fractional (MAbsErr, *M* = .57, *SD* = .17) and familiar units (MAbsErr, *M* = .57, *SD* = .18) tasks, $F(2, 138) = 64.58$, $p < .001$. Age and task also

Table 1
Values presented across estimation tasks (common numerators, familiar units, and decimal units)

Income to be estimated (in decimal units)	Notation presented		
	Common numerators	Familiar units	Decimal units
\$1/min[a]	\$1/1 min	\$1/min	\$1/min
\$0.5000/min	\$1/2 min	\$1/2 min	\$0.5000/min
\$0.1250/min	\$1/8 min	\$1/8 min	\$0.1250/min
\$0.1111/min	\$1/9 min	\$1/9 min	\$0.1111/min
\$0.0167/min	\$1/60 min	\$1/h	\$0.0167/min
\$0.0083/min	\$1/120 min	\$1/2 h	\$0.0083/min
\$0.0021/min	\$1/480 min	\$1/8 h	\$0.0021/min
\$0.0019/min	\$1/540 min	\$1/9 h	\$0.0019/min
\$0.0007/min[a]	\$1/1440 min	\$1/day	\$0.0007/min

[a] End points of the scale.

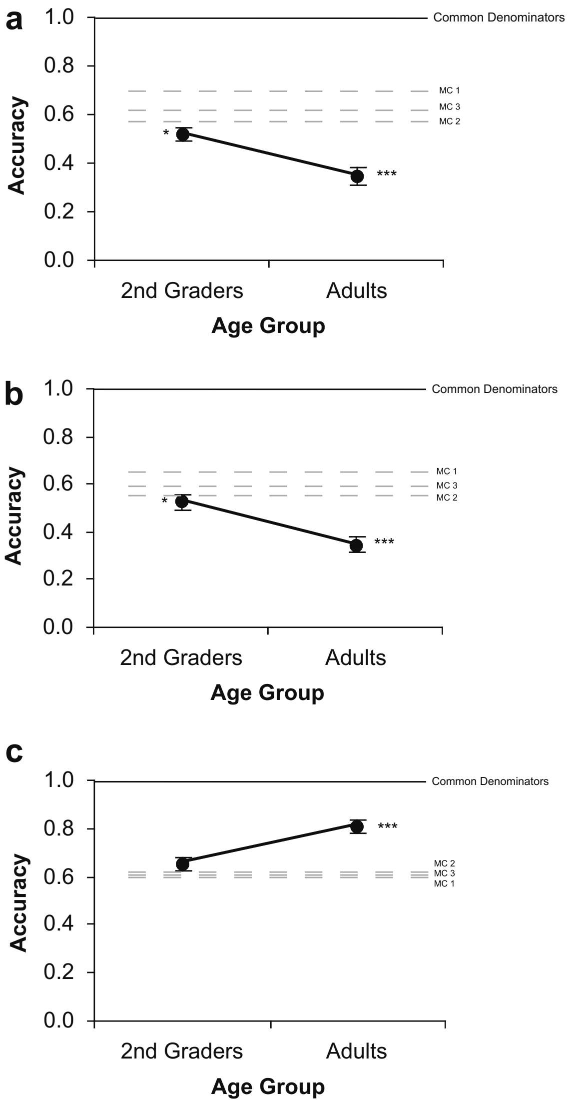

Fig. 2. Accuracy for both age groups on each task: (a) common numerators, (b) familiar units, and (c) decimal units. Dashed lines represent the levels of accuracy obtained in three Monte Carlo simulations (MC 1, MC 2, and MC 3). Asterisks represent the number of simulations differing reliably ($p < .05$) from observed responses.

produced interactive effects on accuracy, $F(2, 138) = 21.0$, $p < .001$, leading us to analyze each task separately.

On the *common numerators task,* adults and children were less accurate ($p < .05$) than random responding (modeled in three Monte Carlo simulations), and adults' overall errors (MAbsErr, $M = 0.65$, $SD = 0.16$) were even greater than children's (MAbsErr, $M = 0.48$, $SD = 0.13$), $t(46) = 4.16$, $p < .001$, $d = 1.23$. To determine whether the large errors at both ages came from ignoring fractional context, we compared estimates on the present 1/1–1/1400 fraction line to those on a 0–1000 number line used by Siegler and Opfer (2003). As Fig. 3 illustrates, the fractional context had no effect on estimates, whereas the value of the denominator had a large effect on estimates. To examine this relation, we regressed magnitude estimates against the value of the denominator using logarithmic and linear regression functions. As predicted, the logarithmic function ($R^2 = .99$) provided a better fit to children's estimates than to all three sets of randomly generated data (average $\log R^2 = .18$), and the logarithmic function provided a better fit than did the linear function ($R^2 = .75$). In contrast, the linear function ($R^2 = .94$) provided a better fit to adults' estimates than to randomly generated data ($\text{lin} R^2 = .12$), and the linear function provided a slightly better fit than did the logarithmic function ($\log R^2 = .90$).

To determine the link between the fit of the two functions and overall accuracy, we fit each function to individual participants' estimates and compared the performance of participants (regardless of age) who used a logarithmic versus linear representation of the denominator (Because both models had an equal number of free parameters, subjects were categorized based on the model producing the greater R^2 value). As expected, the linear group (71% adults) was substantially less accurate (MAbsErr, $M = .68$, $SD = .13$) than the logarithmic group (41% adults) (MAbsErr, $M = .52$, $SD = .13$), $t(46) = 3.28$, $p < .001$, $d = .97$.

The *familiar units task* was examined next to determine whether adults' familiarity with units such as hour, day, and minute might improve their performance. Rather, adults' performance was similar to performance on the common numerators task (Fig. 4), suggesting they did not capitalize on the semantic knowledge the task offered. As in the common numerators task, adults and children were less accurate than random responding (modeled in three Monte Carlo simulations), but adults' overall errors (MAbsErr, $M = 0.66$, $SD = 0.16$) were again much greater than children's (MAbsErr, $M = 0.47$, $SD = 0.16$), $t(46) = 3.97$, $p < .001$, $d = 1.17$. To determine whether adults' poor performance on this task resulted from their translating the day and hour units to minute units (thereby rendering the task identical to the first one), we regressed their median estimates against the value of the denominator in minutes. As predicted, the fit of the linear function to adults' estimates ($R^2 = .99$) was better than the fit to random data (average $R^2 = .19$), and the linear function also provided a better fit than the logarithmic function ($R^2 = .83$), with 67% of adults' estimates being better fit by the linear than logarithmic function. Due to the difficulty of the computational estimation involved, children were not expected to translate familiar units into minutes as well as adults did. Consistent with this expectation, regressing the number of minutes against the median estimate resulted in a logarithmic function ($R^2 = .82$) that did not fit children's estimates as in the common numerators task (cf. Figs. 3 and 4), though the fit of the logarithmic function was still better than the fit of the same function to randomly generated data (average $\log R^2 = .18$). Last, we compared the performance of those participants (regardless of age) who used a linear representation of the denominator to those who used a logarithmic representation, and we found that participants who used a logarithmic representation still provided substantially fewer errors (MAbsErr, $M = .48$, $SD = .16$) than those who used a linear one (MAbsError, $M = .64$, $SD = .17$), $t(46) = 3.50$, $p < .001$, $d = 1.03$.

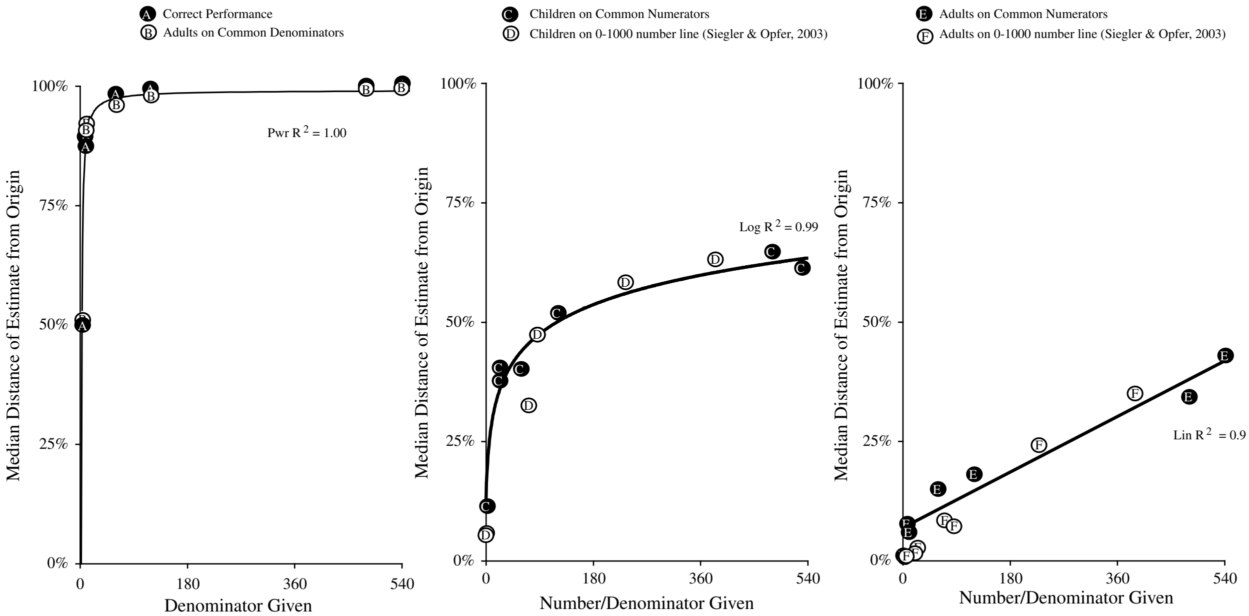

Fig. 3. Fractional units task. Left panel: Adults' estimates on common denominators problems (series B) were best fit by a power function (R^2 = 1.00) and nearly identical to accurate performance (series A). Middle panel: Children's estimates on common numerator problems (series C) were nearly identical to estimates on a 0–1000 number line (series D) from Siegler and Opfer (2003) and best fit by a logarithmic function (R^2 = .99). Right panel: Adults' estimates on common numerator problems (series E) were also nearly identical to adults' performance on a 0–1000 number line (series F) from Siegler and Opfer (2003) and were best fit by a linear function (R^2 = .94).

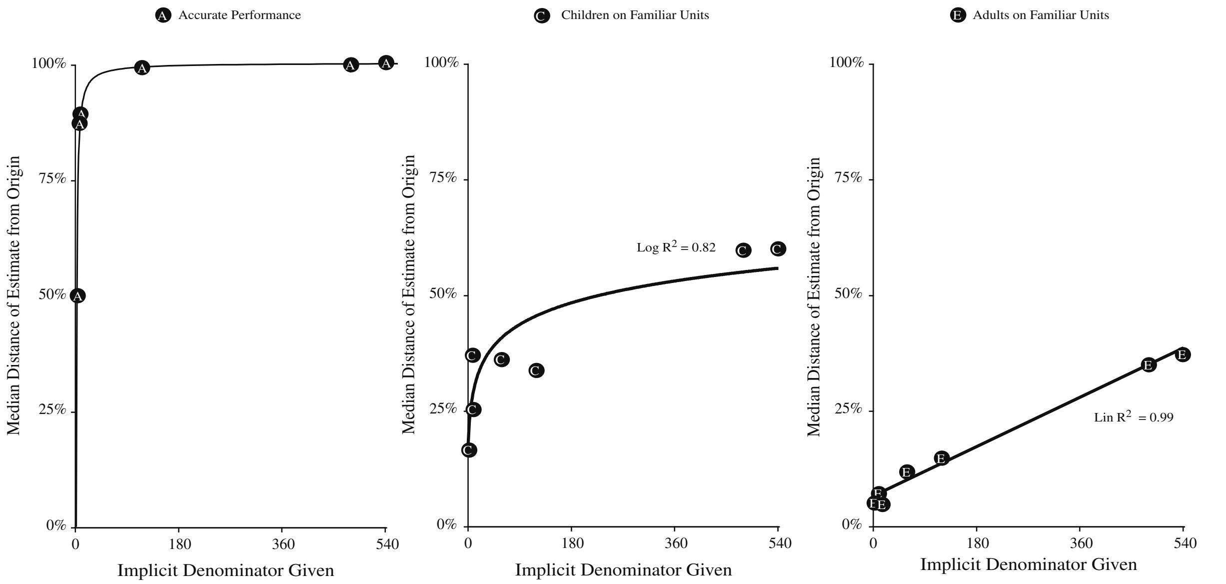

Fig. 4. Familiar units task. Left panel: Accurate performance on the familiar units task (series A). Middle panel: Children's estimates on familiar units problems (series C) were best fit by a logarithmic function (R^2 = .82). Right panel: Adults' estimates on familiar units problems (series E) were best fit by a linear function (R^2 = .99).

Next, we examined performance on a *decimal units task*, which only had one difference between the previous two tasks: the fractional notation, which was the variable we hypothesized to elicit their superior performance. On the decimal units task, children's overall errors (MAbsErr, M = 0.35, SD = 0.13) were indeed much greater than adults' (MAbsErr, M = 0.19, SD = 0.15), $t(46)$ = 4.03, p < .001, d = 1.19. Adults' accuracy was also better than that of randomly generated responses, whereas children's accuracy was not. To determine whether adults' superior performance on this task resulted from their hypothesized use of a linear representation of the decimal units, we regressed their median estimates against actual value. As predicted, the fit of the linear function ($R^2 = .92$) was better than the fit of the logarithmic function ($R^2 = .86$), and the linear function also provided a better fit to adults' estimates than to random responses (average $R^2 = .51$). We also regressed children's median estimates against the value of decimal units, on the assumption that children interpreted decimal values as analogous to whole numbers. As predicted, the fit of the logarithmic function ($R^2 = .54$) was slightly better than the fit of the linear function ($R^2 = .51$); (Fig. 5) clearly neither function provided a very good fit, with only the logarithmic providing a better fit to children's estimates than to randomly generated data (average $\log R^2 = .38$). Finally, when we compared the performance of those participants (regardless of age) who used a linear representation of the decimal versus those who used a logarithmic representation, participants who used a linear representation provided substantially fewer overall errors (MAbsErr, $M = .19$, $SD = .15$) than those who used a logarithmic one (MAbsErr, $M = .34$, $SD = .13$), $t(46) = 3.66$, $p < .001$, $d = 1.08$.

Finally, we examined performance on the common denominators task, which provided a control condition where adults' linear representation of number would be advantageous. As expected, adults' estimates were almost perfectly accurate (MAbsErr, $M = .01$, $SD = .01$), increased linearly as a function of the numerator ($\text{lin}\,R^2 = 1.00$) and on 100% of trials, and increased as a power function of the denominator ($\text{pwr}\,R^2 = 1.00$). Adults' median responses on this task are placed in Fig. 3 (left panel) for comparison to the common numerators task.

While the above analyses classified individual participants as either linear or logarithmic based on their overall performance in each task, participants may have used a different representation on different trials within a task (e.g., Holyoak, 1978). To test this, we used an analysis performed by Geary, Hoard, Byrd-Craven, Nugent, and Numtee (2007) where we looked at each individual trial and classified each trial as linear, logarithmic or ambiguous. Each response was compared to the predicted response based on both a linear and a logarithmic function that was constrained to fit between the end points of the number line. The function that each response was closer to defined the trial as linear or logarithmic. Any response within 5% of both the linear and logarithmic function was classified as ambiguous. The results of this analysis are in Table 2.

4. General discussion

We tested the claim that age-related improvements in estimation are caused by the development of linear representations. Previous support for this hypothesis came from age-differences in performance on tasks in which a linear representation necessarily yields accurate estimates (Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003), which has left it unclear whether the development of representations (Joram et al., 1998) or improvements in general mathematical skills could be responsible for estimation improvement (Hiebert & Wearne, 1986; Noël et al., 2005). In this study, we sought to provide a particularly strong test for the representational change hypothesis by examining development on an estimation task that favors the logarithmic representation at the expense of the linear one (i.e., estimation of fractional magnitudes). An interesting prediction generated by the representational change hypothesis was that children would outperform adults when asked to make estimates of fractional magnitudes (e.g., comparing 1/100 to 1/1 and 1/1000).

The results of the present study supported this hypothesis. When estimating the value of a salary by estimating its place on a scale expressed in fractional notation (i.e.,

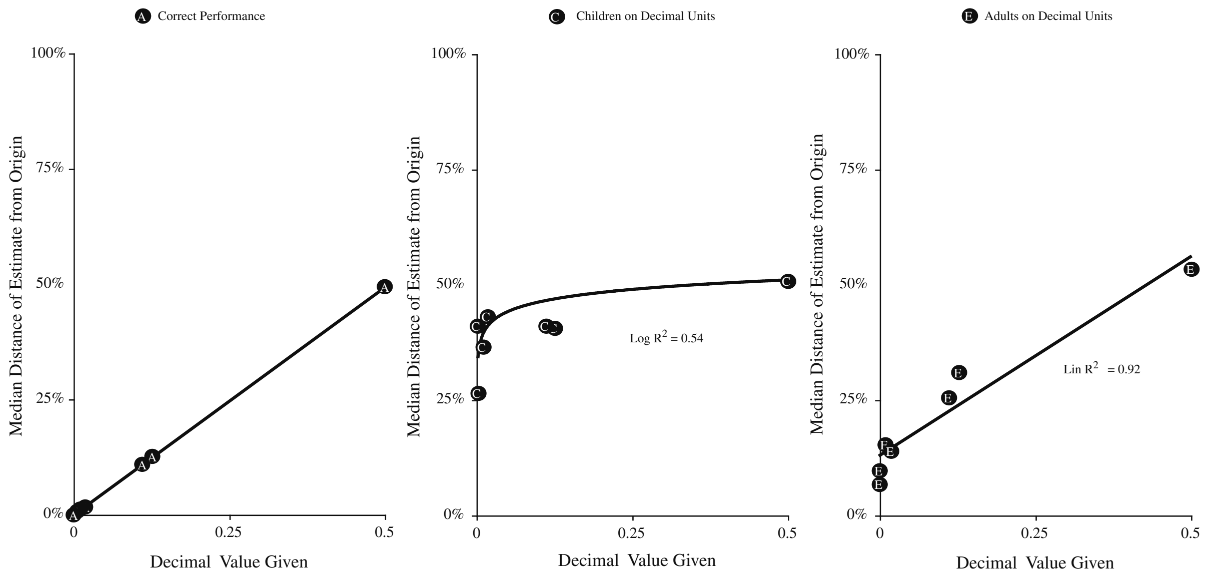

Fig. 5. The decimal units task. Left panel: Accurate performance on the decimal units task (series A). Middle panel: Children's performance on the decimal units task (series C) was less accurate than adults' performance on the decimal task and was modestly fit by a logarithmic function ($R^2 = .54$). Right panel: Adults' performance on the decimal units task (series E) was highly accurate and best fit by a linear function ($R^2 = .92$).

Table 2
Results

	Mean absolute error	Best R^2 fit	% Best fit by lin, adults vs. children	% Trials best fit by lin function	% Trials best fit by log function
Common denominators					
Adults	0.01	Lin $R^2 = 1.00$	n/a	84	0
Common numerators					
Adults	0.65*	Lin $R^2 = .94$	Adults (46%) >	76	36
Children	0.48*	Log $R^2 = .99$	Children (17%)*	21	60
Familiar units					
Adults	0.66*	Lin $R^2 = .99$	Adults (67%) >	77	36
Children	0.47*	Log $R^2 = .82$	Children (38%)*	22	60
Decimal units					
Adults	0.19*	Lin $R^2 = .92$	Adults (73%) >	50	33
Children	0.35*	Log $R^2 = .54$	Children (46%)*	40	66

An asterisk represents significance with an alpha of 0.05 or lower. Chi-square tests were used for comparisons of fit, and planned pair-wise *t*-tests were used to compare mean absolute error.

on a scale where a logarithmic mental scaling of salaries provided a more accurate gauge than a linear mental scaling), children provided more accurate estimates than adults. Clearly this difference did not stem from children having greater mathematical skills than adults. Rather, it appears that children used a logarithmic representation of the denominator's value as a guide for placing their estimates, whereas adults used a linear representation of the denominator's value as a guide for placing their estimates. The clearest evidence for this hypothesis came from a comparison of estimates on our 1/1–1/1440 task and Siegler and Opfer's (2003) 0–1000 task: as Fig. 3 illustrates, the two sets of estimates are nearly indistinguishable, with the fit of the logarithmic function being very high in both tasks for children (this study, log $R^2 = .99$; S&O, log $R^2 = .95$) and the fit of the linear function being very high in both tasks for adults (this study, lin $R^2 = .94$, S&O, lin $R^2 = 1$).

More broadly, we believe our unusual developmental findings illustrate an important principle: alternative hypotheses are most usefully distinguished when they are tested against events with a low base-rate of probability. We believe that this principle is a particularly important one for making causal inferences in the development of mathematical cognition and in developmental psychology more broadly, where everything seems to improve with age. On this basis, we believe we have provided strong evidence for children and adults possessing different representations of numerical magnitude.

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